Convex Optimization Techniques for Interference Mitigation in Cognitive Radio Networks

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Convex Optimization and Interference Mitigation Techniques

1. Introduction to Convex Optimization Techniques

2. Applications in Spectrum Sharing Wireless Channels
   (Coexistence of PU and SU network)

   - Beamforming Techniques for Interference Mitigation

   - Cognitive Relays in Overlay Networks

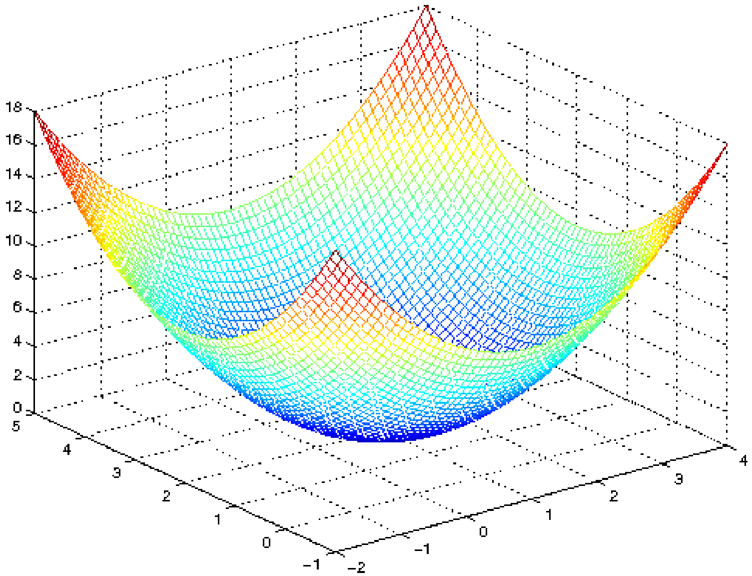
   - MIMO Cognitive Radio Networks

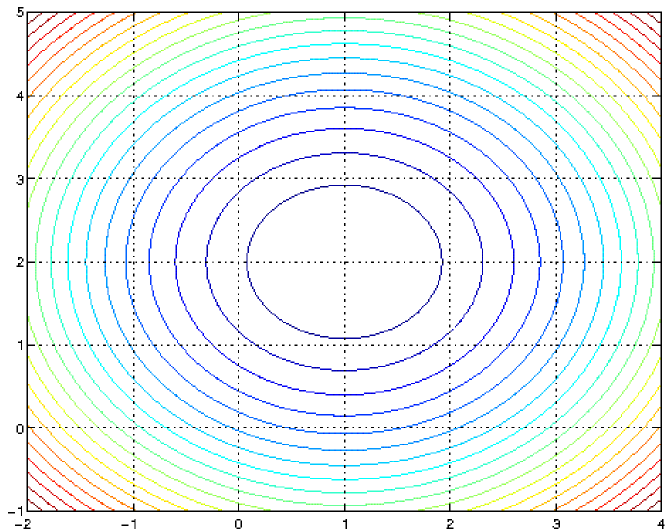
   - Interference Control in OFDMA

   - Non-Cooperative Robust Rate Maximization Games in OFDM

Example of an unconstrained optimization problem

Minimise $J = (x_1 - 1)^2 + (x_2 - 2)^2 = (\mathbf{x} - \mathbf{x}_c)^T (\mathbf{x} - \mathbf{x}_c)$

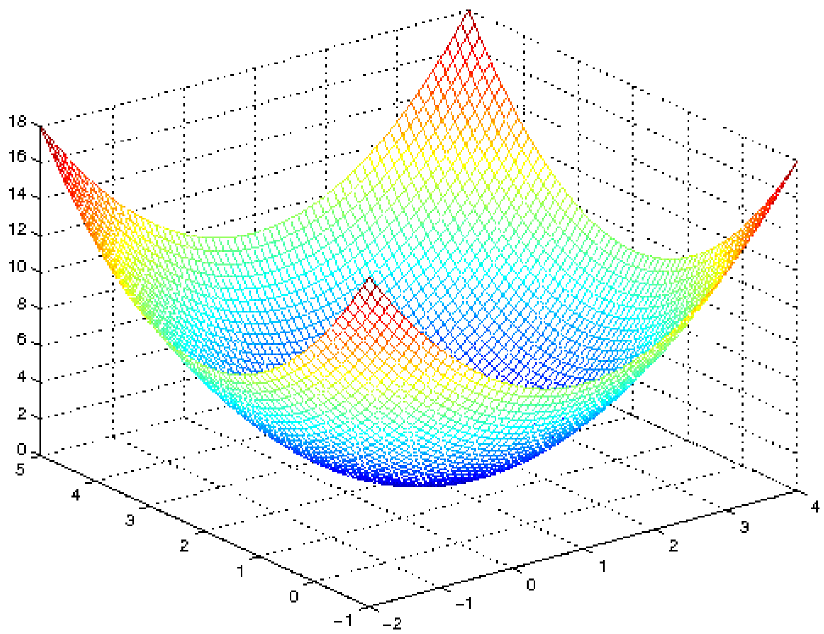


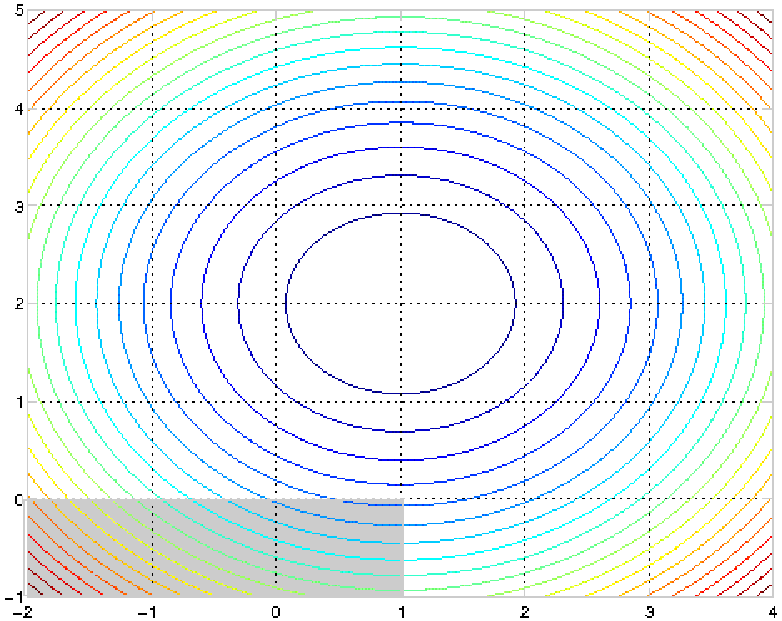


Examples of constrained optimization problems

Minimise $J = (x_1 - 1)^2 + (x_2 - 2)^2 = (\mathbf{x} - \mathbf{x}_c)^T (\mathbf{x} - \mathbf{x}_c)$

subject to
$$x_1 \leq 1$$
$$x_2 \leq 0$$

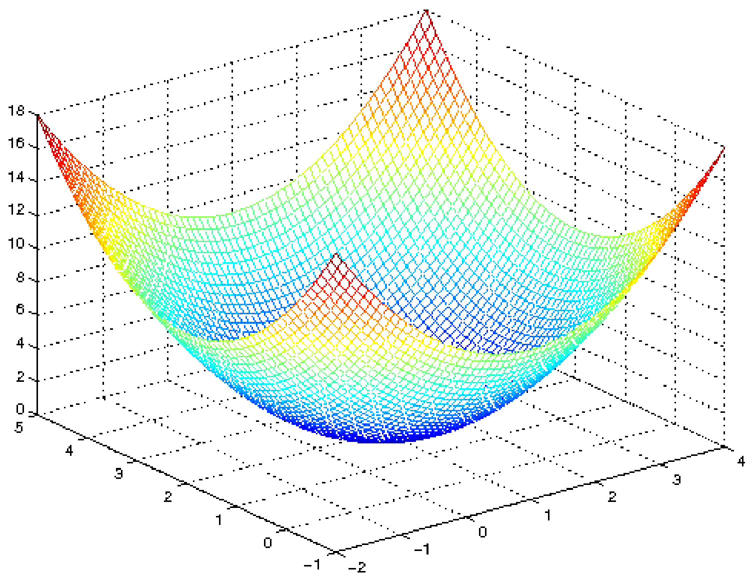


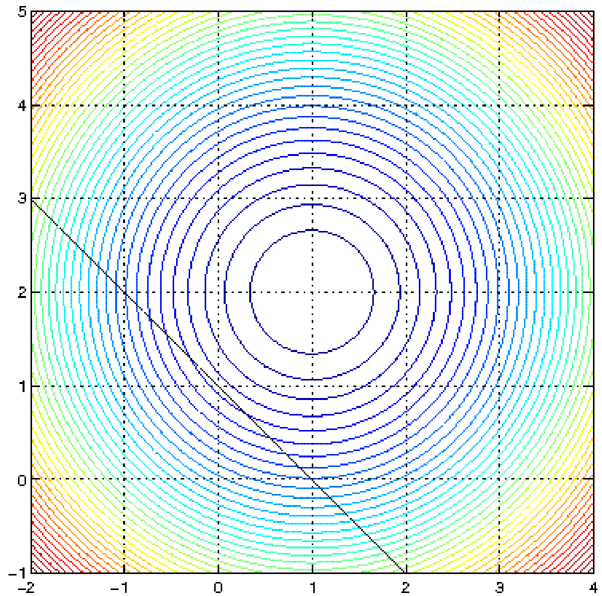


Examples of constrained optimization problems

Minimise $J = (x_1 - 1)^2 + (x_2 - 2)^2 = (\mathbf{x} - \mathbf{x}_c)^T (\mathbf{x} - \mathbf{x}_c)$

subject to $a^T (x - x_0) \le 0$ where $a = [1 \quad 1]^T$ and $x_0 = [1 \quad 0]^T$





$$L(\mathbf{x}, \lambda) = (\mathbf{x} - \mathbf{x}_c)^T (\mathbf{x} - \mathbf{x}_c) + \lambda\, \mathbf{a}^T (\mathbf{x} - \mathbf{x}_0)$$

$$\frac{\partial L(\mathbf{x}, \lambda)}{\partial \mathbf{x}} = 2\mathbf{x} - 2\mathbf{x}_c + \lambda \mathbf{a} = 0 \quad \Rightarrow \mathbf{x}_{opt} = \mathbf{x}_c - \frac{\lambda}{2}\mathbf{a}$$

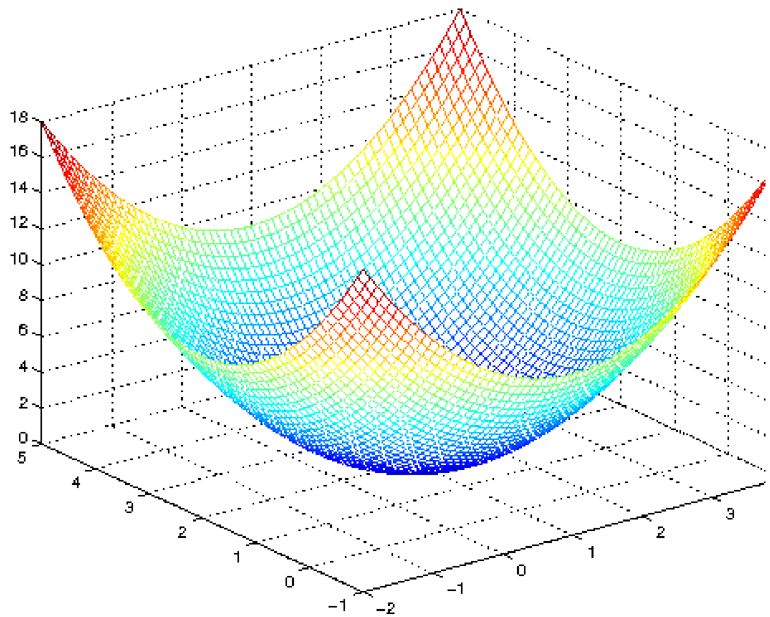
$$\mathbf{a}^T (\mathbf{x}_{opt} - \mathbf{x}_0) = 0 \quad \Rightarrow \quad \mathbf{a}^T (\mathbf{x}_c - \frac{\lambda}{2}\mathbf{a} - \mathbf{x}_0) = 0 \quad \Rightarrow \quad \lambda = 2$$

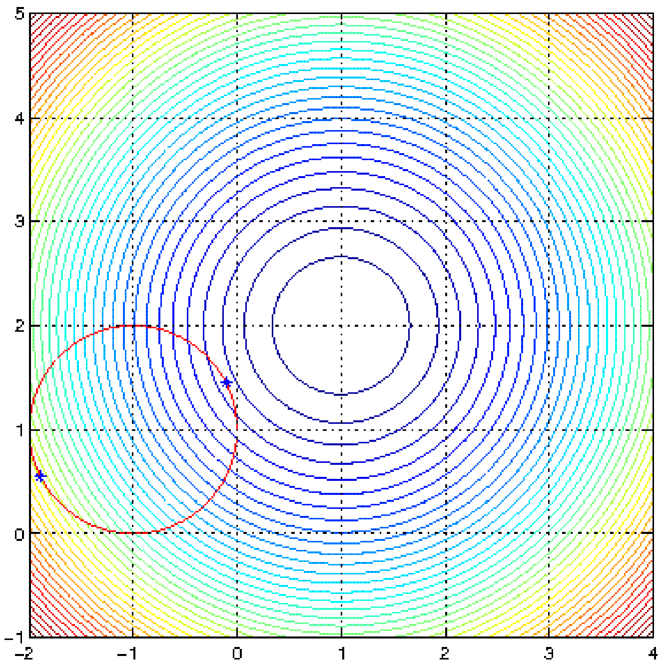
Hence $\quad \mathbf{x}_{opt} = \mathbf{x}_c - \frac{\lambda}{2}\mathbf{a} = \mathbf{x}_c - \mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Examples of constrained optimization problems

Minimise $J = (x_1 - 1)^2 + (x_2 - 2)^2 = (\mathbf{x} - \mathbf{x}_c)^T (\mathbf{x} - \mathbf{x}_c)$

subject to $\left\| (x - x_\mu) \right\|_2^2 \leq 1$ where $x_\mu = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$





Convex Optimization Problem

$$\begin{aligned} &\text{minimise} && f_0(x) \\ &\text{subject to} && f_i(x) \le 0 \quad i = 1,2,\cdots,m \\ & && h_i(x) = b_i \quad i = 1,2,\cdots,p \end{aligned}$$

The objective function must be convex.
Inequality constraint functions must be convex.
The equality constraint function must be affine.

The feasible set of a convex optimization problem is convex.

$$D = \bigcap_{i=0}^{m} dom\ f_i$$

i.e. A convex objective function is minimised over a convex set.

- Linear Programming (LP)
- Linear Fractional Programming
- Quadratic Programming (QP)
- Quadratically Constrained Quadratic Programming (QCQP)
- Second Order Cone Programming (SCP)
- Geometric Programming (GP)
- Semidefinite Programming (SDP)

Linear Programming (LP)

$$\begin{aligned} \text{minimise} \quad & c^T x \\ \text{subject to} \quad & G\,x \le h \\ & A\,x = b \end{aligned}$$

Linear Fractional Programming (LP)

minimise $f_0(x)$

subject to
$$G\,x \le h$$
$$A\,x = b$$

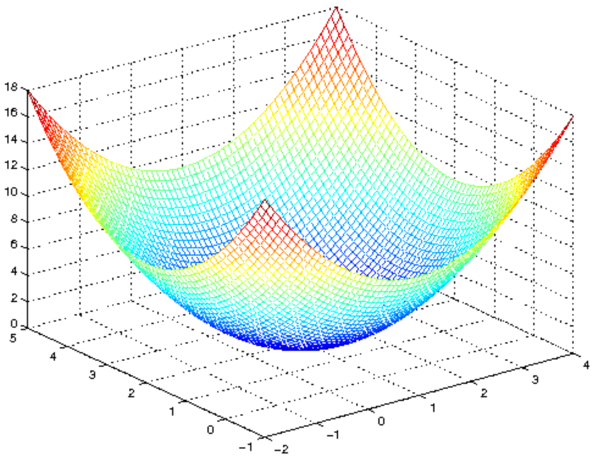
where $f_0(x) = \dfrac{c^T x + d}{e^T x + f}$ $\quad dom\ f_0 = \{x \mid e^T x + f > 0\}$

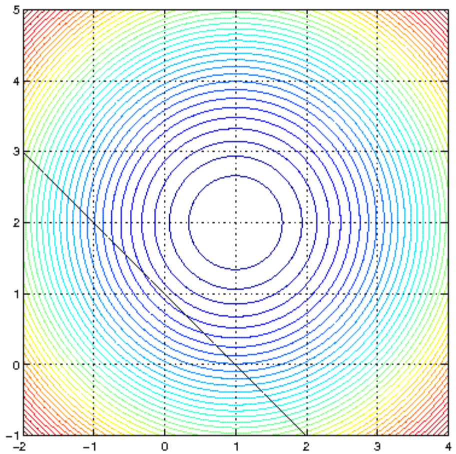
Quadratic Programming

$$\text{minimise} \quad x^T P x + q^T x + r$$

$$\text{subject to} \quad \begin{array}{l} G x \leq h \\ A x = b \end{array}$$

where $P \in S_+^n, G \in R^{m\mathrm{x}n}, A \in R^{p\mathrm{x}n}$

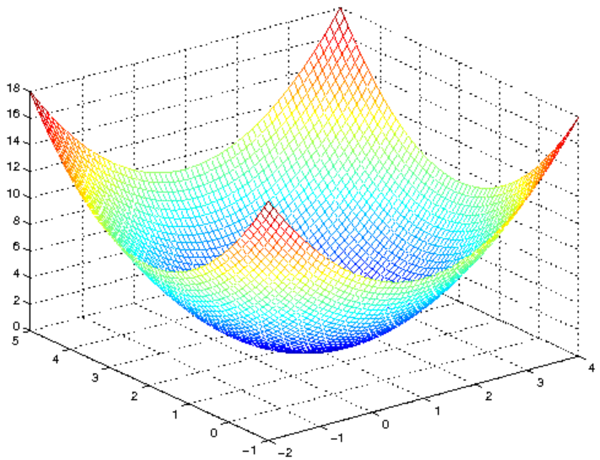


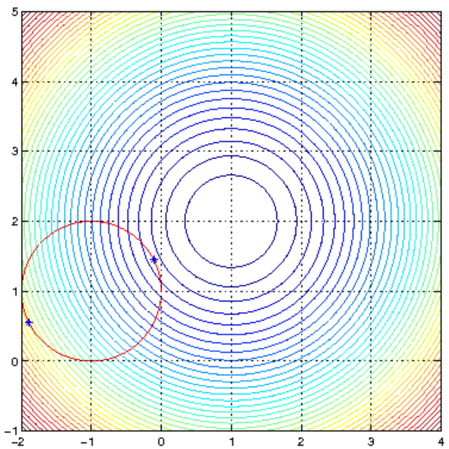


Quadratically Constrained Quadratic Programming

$$\begin{array}{ll} \text{minimise} & x^T P_0 x + q_0^T x + r_0 \\ \text{subject to} & x^T P_i x + q_i^T x + r_i \le 0 \quad i = 1, 2, \cdots, m \\ & A x = b \end{array}$$

where $P \in S_+^n, \quad A \in R^{p\text{x}n}$





Second Order Cone Programming (SP)

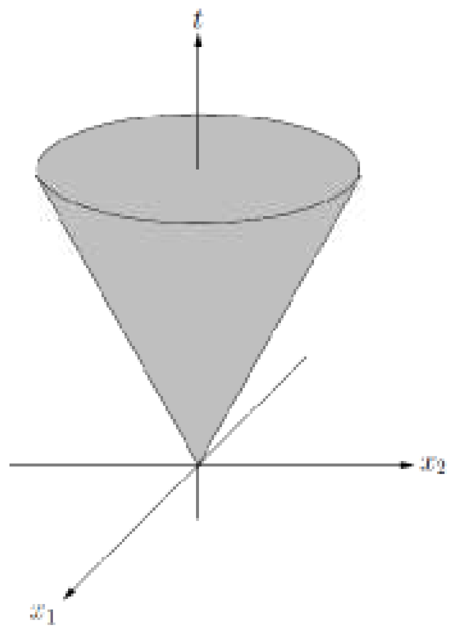
minimise $f^T x$

subject to $\|A_i x + b_i\|_2 \le c_i^T x + d_i \quad i = 1,2,\cdots,m$

$$F x = g$$

where $x \in R^n$, $A_i \in R^{n_i x n}$, $F \in R^{p x n}$

The constraint $\|A_i x + b_i\|_2 \le c_i^T x + d_i$ is called a second order cone constraint.



Second-order cone in $\mathbf{R}^3$, $\left\{(x_1, x_2, t) \mid (x_1^2 + x_2^2)^{1/2} \le t\right\}$

Source: Stephen Boyd- Convex Optimization

Geometric Programming

$$\begin{aligned} \text{minimise} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le 1 \quad i = 1,2,\cdots,m \\ & h_i(x) = 1 \quad i = 1,2,\cdots,p \end{aligned}$$

where $f_i(x), \quad i = 0,1,2,\cdots,m$ are posynomials and

$h_i(x) = 1 \quad i = 1,2,\cdots,p$ are monomials

Domain of the problem $D = R_{++}^{n}$

Monomial function: $h(x) = c\, x_1^{a1}\, x_2^{a2} \cdots x_n^{an}, \quad c > 0, a_i \in R$

Posynomial function: $f(x) = \sum_{k=1}^{K} c_k\, x_1^{a_{1k}}\, x_2^{a_{2k}} \cdots x_n^{a_{nk}}, \quad c_i > 0, a_{ik} \in R$

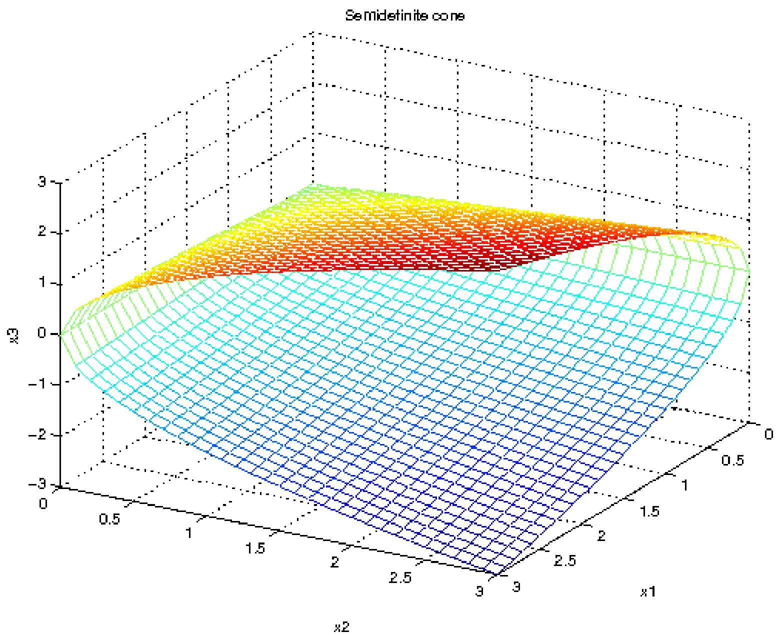
Semidefinite Programming

minimise $c^T x$

subject to $x_1 F_1 + x_2 F_2 + \cdots + x_n F_n + G \leq 0$
$A\,x = b$

where $F_1, F_2, F_n, G \in S^k$, $A \in R^{p\text{x}n}$

An example of semidefinite cone $\mathbf{X} = \begin{bmatrix} x1 & x3 \\ x3 & x2 \end{bmatrix} \geq 0$



Cognitive Radio Network

❑A spectrum crisis for future and emerging wireless services as wide portions of the available radio spectrum have already been allocated to various services.

❑Various spectrum occupancy measurements demonstrate that large portions of the spectral bands are unoccupied most of the time.

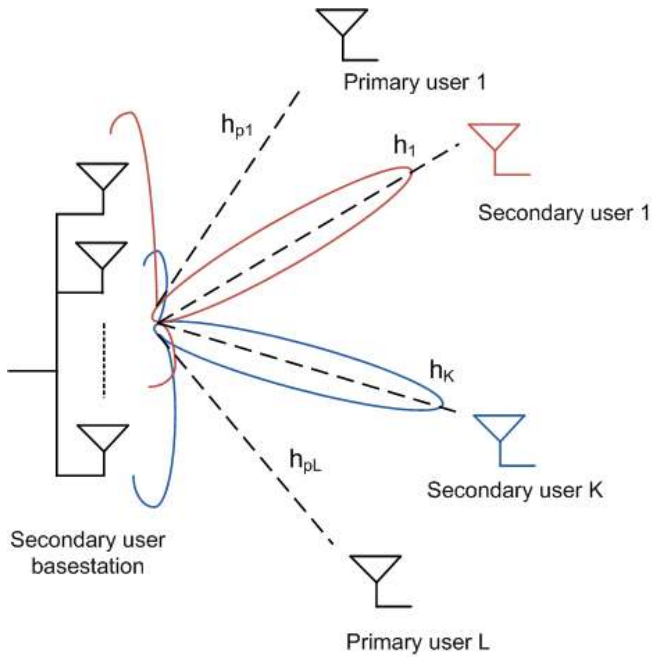
❑Cognitive radios has the ability to sense the environment for availability of spectrum and adjust its transmission parameters.

❑Term first coined by Joseph Mitola in “Cognitive Radio: Making Software Radios More Personal,” IEEE Personal Communications, Aug. 1999

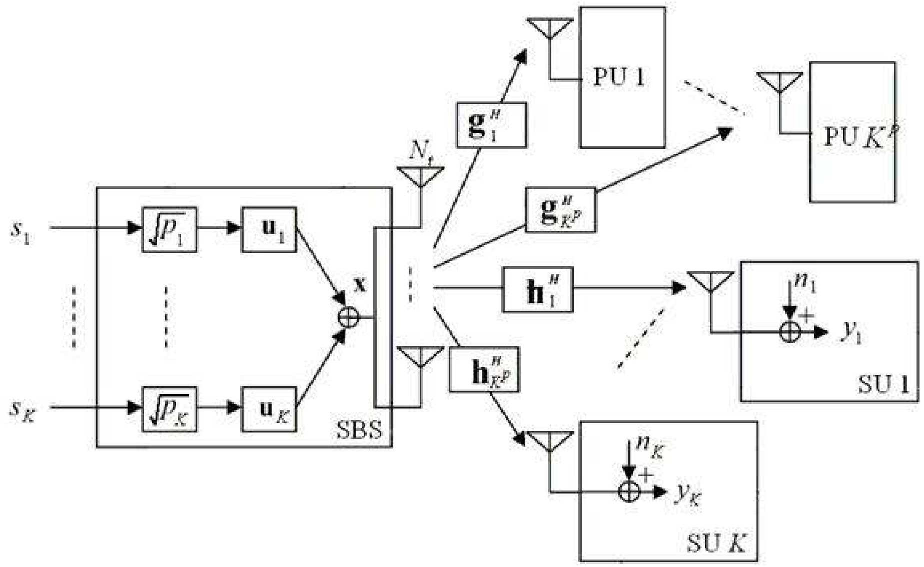
❑Three types of CR networks: Overlay, underlay and interweaved networks.

Application of Convex Optimization in Cognitive Radio Networks

- ❑Interference mitigation using beamforming techniques
- ❑SINR balancing based quality of service provision.
- ❑Cognitive relaying in overlay networks.
- ❑Cognitive MIMO networks
- ❑Adaptive bit loading and power allocation for OFDMA based CR networks.
- ❑ Robust rate maximization games in spectrum sharing channels.



Interference Mitigation Using Beamforming



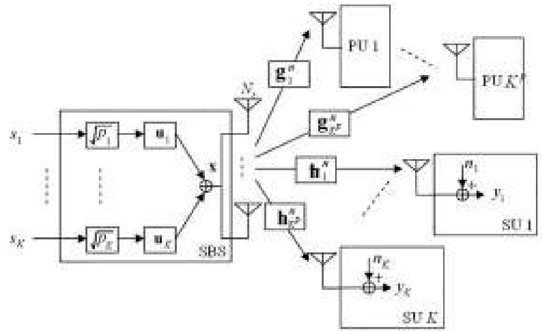
- ❑ A downlink CR network with K SUs and L PUs.
- ❑ SU basestation has multiple antennas and each PU and SU has single antenna.

The transmitted signal from the basestation is $\mathbf{x} = \sum_{i=1}^{K} \sqrt{p_i}\, \mathbf{u}_i s_i = \tilde{\mathbf{U}}\mathbf{s}$

The aim is to determine an optimum set of beamformers and power allocations so that each SU attains its target signal to interference plus noise ratio while ensuring interference leakage to primary users is below a certain threshold.

The SINR at the ith SU terminal is $\mathrm{SINR_i} = \dfrac{\tilde{\mathbf{u}}_i^H \mathbf{R}_i \tilde{\mathbf{u}}_i}{\sum_{\mathbf{j}\neq\mathbf{i}} \tilde{\mathbf{u}}_j^H \mathbf{R}_i \tilde{\mathbf{u}}_j + \sigma_i^2}$

Constrained Optimization Framework



The optimization problem is stated as minimise "the total transmitted power subject to the following three set of constraints":

1) Each secondary user attains a required SINR target.
2) The interference leakage to each primary user is below a threshold.
3) The total transmission power at the basestation is limited.

minimise $\sum_{\mathbf{k}} \tilde{\mathbf{u}}_k^H \tilde{\mathbf{u}}_k$

subject to $$\frac{\tilde{\mathbf{u}}_i^H \mathbf{R}_i \tilde{\mathbf{u}}_i}{\sum_{\mathbf{k}\neq\mathbf{i}} \tilde{\mathbf{u}}_k^H \mathbf{R}_i \tilde{\mathbf{u}}_k + \sigma_i^2} \geq \rho \qquad i = 1,2,\dots,K$$

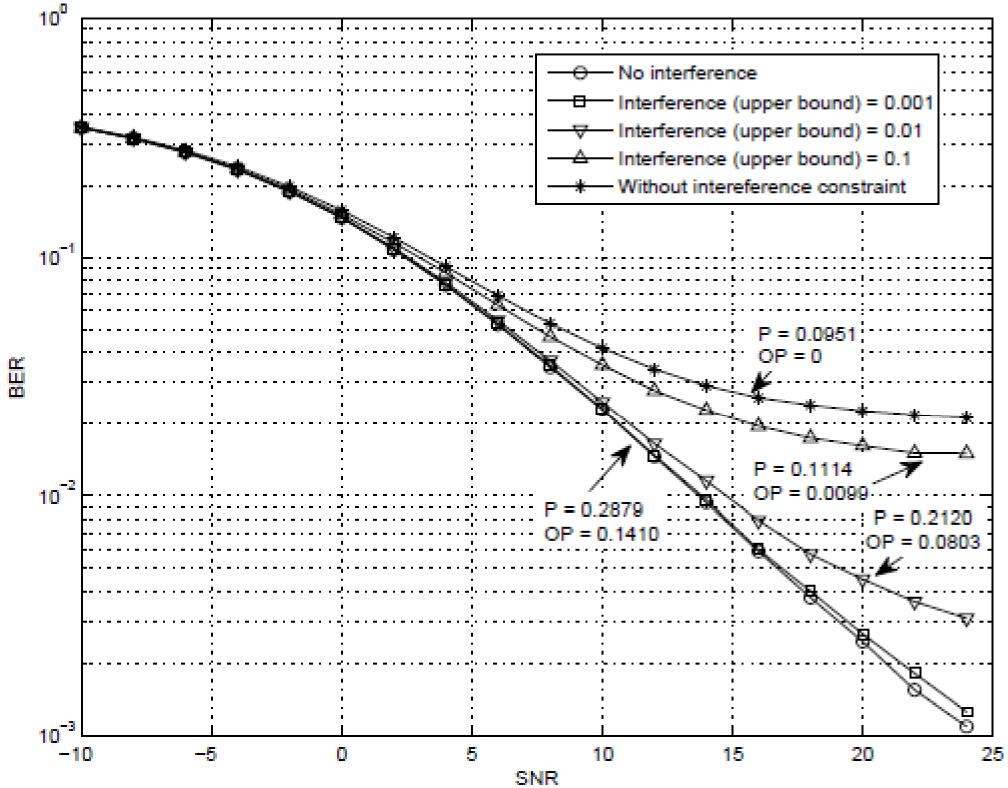
$$\mathbf{1}^T \mathbf{p} \leq P_{MAX},$$

$$\sum_{i=1}^{K} \tilde{\mathbf{u}}_i^H \mathbf{G}_j \tilde{\mathbf{u}}_i \leq \beta_j \qquad j = 1,2,\dots,L$$

The above problem can be solved using second order programming for rank-1 matrices $\mathbf{R}_i$ and using semidefinite programming (using Lagrangian relaxation) for general rank matrices $\mathbf{R}_i$.

For higher SINR targets, the problem might turn out to be infeasible. Another possible formulation of the problem is to maximise the worst case user SINR subject to the limited transmission power.

Simulation Result



The BER performance of the PU for different upper bounds on the interference temperatures. The average transmitted powers at the SNBS and the outage probabilities are shown next to the corresponding graphs. The target SINR of the SU is 10 dB.

SINR Balancing Based Beamformer

For higher SINR targets, the problem might turn out to be infeasible. Another possible formulation of the problem is to maximise the worst case user SINR subject to the limited transmission power.

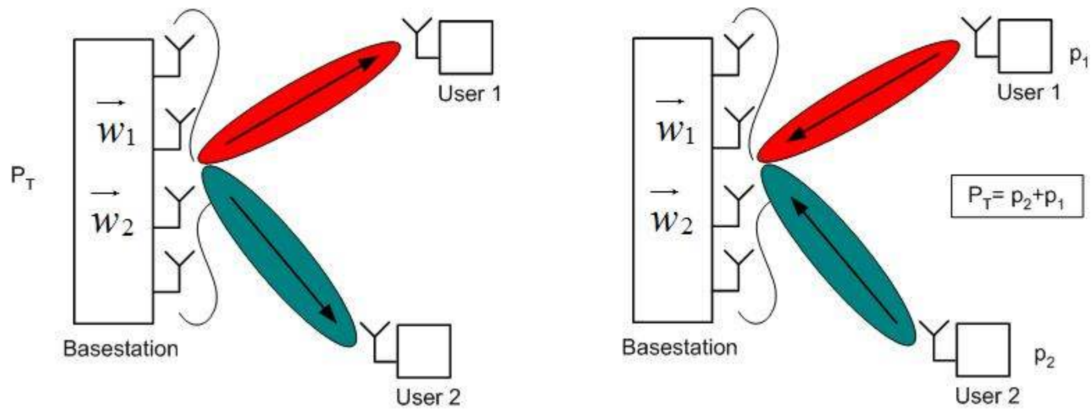
K. Cumanan, L. Musavian, S. Lambotharan and A. Gershman, "SINR Balancing Technique for Downlink Beamforming in Cognitive Radio Networks," IEEE Signal Processing Letters, vol. 17, pp. 133-136, Feb. 2010.

The aim is to maximise the worst case SINR for all users (i.e. SINR balancing) subject to constraints on transmission power and interference leakage as follows,

$$\max_{\mathbf{U},\mathbf{p}} \min_{1 \le i \le K} SINR_i(\mathbf{U},\mathbf{p})$$

subject to $\mathbf{1}^T\mathbf{p} \le P_{MAX}$ $\quad p_i \left\| \mathbf{g}_j^H \mathbf{u}_i \right\|_2^2 \le \beta_{i,j}$ $\quad i = 1,2,\dots,L$ and $j = 1,2,\dots,K$

We adopt the duality theory developed for uplink-downlink beamforming in Schubert & Boche, 2004



Both the uplink and the downlink share the same SINR region under total power constraint.

Use of Uplink and Downlink Duality

- In Schubert&Boche, 2004, the beamformers designed in the virtual uplink (with power allocation q_k) can be used in the downlink with a different power allocation p_k , to achieve same SINR region.
- This can not be applied to cognitive radio network because interference leakage in the downlink should be considered when beamformers are designed in the virtual uplink.
- Hence we modify the methods in Schubert&Boche, 2004 to design the virtual uplink beamformers while explicitly considering interference leakage to PUs as follows

$$\max_{\mathbf{u}_j} \frac{\mathbf{u}_j^H \widetilde{\mathbf{R}}_j \mathbf{u}_j}{\mathbf{u}_j^H \widetilde{\mathbf{Q}}_j \mathbf{u}_j}$$

subject to $p_i \left\| \mathbf{g}_j^H \mathbf{u}_i \right\|_2^2 \leq \beta_{i,j} \quad i = 1,2,\dots,K_P, \ \|\mathbf{u}_i\|_2^2 = 1$

where $\widetilde{\mathbf{Q}}_k = \sum_{i=1,i\neq k}^{K} q_i \frac{\mathbf{R}_i}{\sigma_i^2}$ and $\widetilde{\mathbf{R}}_k = \frac{\mathbf{R}_k}{\sigma_i^2}$

Converting into Semidefinite Programming

Introducing a new variable $\mathbf{w}_j = \widetilde{\mathbf{Q}}_j^{1/2}\mathbf{u}_j$, and setting $\|\mathbf{w}_i\|_2^2 = 1$, we could write the above optimization as

$$\max_{\mathbf{w}_j} \quad \mathbf{w}_j^H \widetilde{\mathbf{Q}}_j^{-1/2} \widetilde{\mathbf{R}}_j \widetilde{\mathbf{Q}}_j^{-1/2} \mathbf{w}_j$$

$$\text{subject to} \quad p_i \left\| \mathbf{g}_j^H \mathbf{u}_i \right\|_2^2 \leq \beta_{i,j} \quad i = 1,2,\dots,K_P,\ \|\mathbf{w}_i\|_2^2 = 1$$

The interference constraint has been set with an assumption that $\|\mathbf{u}_i\|_2^2 = 1$. A difficulty arises since we can not simultaneously set unity norm constraint for both $\mathbf{u}$ and $\mathbf{w}$. We therefore eliminate the dependency of $\mathbf{u}$ while ensuring $\|\mathbf{u}_i\|_2^2 = 1$ as follows,

$$p_i \left\| \mathbf{g}_j^H \mathbf{u}_i \right\|_2^2 \leq \beta_{i,j}$$

$$\Rightarrow \quad p_i \left\| \mathbf{g}_j^H \frac{\widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} \mathbf{w}_j}{\left\| \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} \mathbf{w}_j \right\|} \right\|_2^2 \leq \beta_{i,j}$$

$$\Rightarrow \quad p_i \frac{\mathbf{w}_j^H \widetilde{\mathbf{Q}}_j^{-1/2} \mathbf{G}_j \widetilde{\mathbf{Q}}_j^{-1/2} \mathbf{w}_j}{\mathbf{w}_j^H \widetilde{\mathbf{Q}}_j^{-1} \mathbf{w}_j} \leq \beta_{i,j}$$

$$\Rightarrow \mathbf{w}_j^H [\, p_i \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} \mathbf{G}_j \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} - \beta_{i,j} \widetilde{\mathbf{Q}}_j^{-1}] \mathbf{w}_j \leq 0$$

Converting into Semidefinite Programming

Defining a rank one matrix as $\mathbf{D_j} = \mathbf{w}_j \mathbf{w}_j^H$, the above problem is converted to Semidefinite programming as follows,

$$\max_{\mathbf{w}_j} Tr\left\{ \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} \widetilde{\mathbf{R}}_j \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} \mathbf{D}_j \right\}$$

subject to

$$Trace\{ [\, p_i\, \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}}\, \mathbf{G}_j\, \widetilde{\mathbf{Q}}_j^{-\frac{1}{2}} - \beta_{i,j}\, \widetilde{\mathbf{Q}}_j^{-1}]\, \mathbf{D}_j \,\} \leq 0, j = 1,2, \dots, K_P$$

$$Trace\left\{ \mathbf{D}_j \right\} = 1, \quad \mathbf{D}_j = \mathbf{D}_\mathbf{j}^{\mathrm{H}}, \quad \mathbf{D}_j \geq 0, \quad \mathrm{rank}\left\{ \mathbf{D}_j \right\} = 1$$

The power allocation method for virtual uplink and downlink modes is similar to that proposed in Schubert & Boche 2004.

Optimality and Results

SU basestation with 6 antennas. Three SUs and two PUs. Total transmit power 5.0. Interference threshold 0.1. Noise power 0.05.

The power allocation for each user and the balanced SINR are obtained for 5 different randomly generated channels.

Channels	User 1 Power (W)	User 2 Power (W)	User 3 Power (W)	Total Power (W)	User 1 SINR (dB)	User 2 SINR (dB)	User 3 SINR (dB)
Channel 1	0.5510	2.3736	2.0755	5	14.1964	14.1964	14.1964
Channel 2	1.7268	1.4452	1.8281	5	20.0236	20.0236	20.0236
Channel 3	1.9553	1.7080	1.3367	5	18.4761	18.4761	18.4761
Channel 4	1.0801	0.6855	3.2344	5	17.5356	17.5356	17.5356
Channel 5	1.8645	1.6597	1.4758	5	17.8571	17.8571	17.8571

The achieved SINRs are set as target SINRs and beamformers are designed using Semidefinite programming to compare the performance. Both yields identical beamformers and power allocations.

Channels	Target SINR (dB)	Maximum available power (W)	User 1 Power (W)	User 2 Power (W)	User 3 Power (W)
Channel 1	14.1964	5	0.5510	2.3736	2.0755
Channel 2	20.0236	5	1.7268	1.4452	1.8281
Channel 3	18.4761	5	1.9553	1.7080	1.3367
Channel 4	17.5356	5	1.0801	0.6855	3.2344
Channel 5	17.8571	5	1.8645	1.6597	1.4758

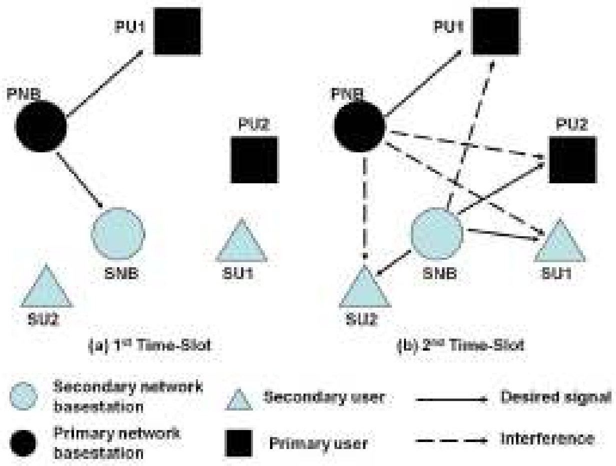
min $\sum_{\mathbf{k}} \tilde{\mathbf{u}}_k^H \tilde{\mathbf{u}}_k$

subject to $\mathbf{1}^T \mathbf{p} \leq P_{MAX}$, $\dfrac{\tilde{\mathbf{u}}_i^H \mathbf{R}_i \tilde{\mathbf{u}}_i}{\sum_{\mathbf{k \neq i}} \tilde{\mathbf{u}}_k^H \mathbf{R}_i \tilde{\mathbf{u}}_k + \sigma_i^2} \geq \rho$

$\|\mathbf{g}_j^H \tilde{\mathbf{u}}_i\|_2^2 \leq \beta_{i,j} \quad i = 1,2,\dots, K_P$ and $j = 1,2,\dots, K$

Beamforming for Overlay Cognitive Radio Network

J. Tang and S. Lambotharan, "Beamforming and Temporal Power Optimization for an Overlay Cognitive Radio Relay Network," provisionally accepted for publication in IET Signal Processing.



Secondary network uses the primary radio spectrum.

In return, the secondary network basestation (SNB) assists the primary network to relay signals to users that are not within the coverage area of the primary network basestation (PNB).

In the first time slot, PNB transmit signals and the SNB listens. In the second time slot, SNB relays the primary user signal while transmitting signal to its users.

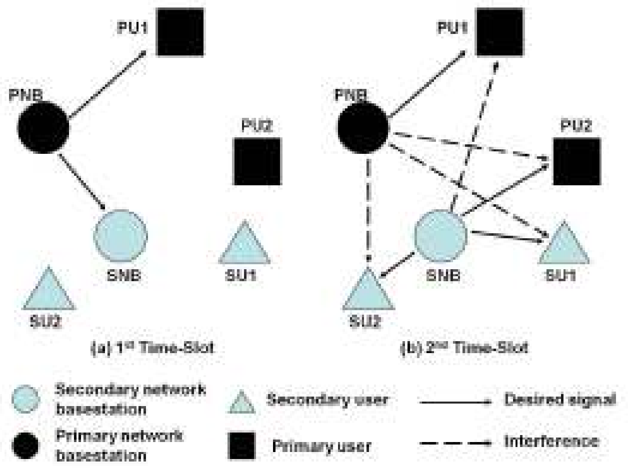
In the first time slot: $y_{p1,1}(n) = \mathbf{w}_{p1,1}^H \mathbf{g}_1 s_{p1}(n) + \mathbf{w}_{ps} s_{p2}(n) \mathbf{g}_1 + q_1(n),$

$$y_{BS}(n) = \mathbf{w}_{ps}^H \mathbf{g}_{ps} s_{p1}(n) + \mathbf{w}_{1,1} \mathbf{g}_{ps} s_{p2}(n) + q_2(n).$$

$$SINR_{PU1} = \frac{\mathbf{w}_{p1,1}^H \mathbf{R}_{p1} \mathbf{w}_{p1,1}}{\mathbf{w}_{ps}^H \mathbf{R}_{p1} \mathbf{w}_{ps} + \sigma_n^2}$$

$$SINR_{SNB} = \frac{\mathbf{w}_{ps}^H \mathbf{R}_{ps} \mathbf{w}_{ps}}{\mathbf{w}_{p1,1}^H \mathbf{R}_{ps} \mathbf{w}_{p1,1} + \sigma_n^2}.$$

Beamforming for Overlay Cognitive Radio Network



In the second time slot:

$$y_{p1,2}(n) = \mathbf{w}_{p1,2}^H \mathbf{g}_1 s_{p1,2}(n) + \sum_{i=1}^{K+1} \mathbf{w}_i \mathbf{h}_{p1} s_k(n) + q_3(n)$$

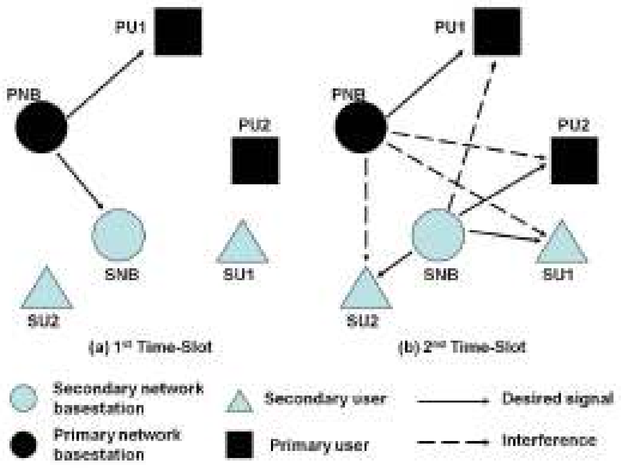
$$y_{sk}(n) = \mathbf{w}_k^H \mathbf{h}_k s_k(n) + \mathbf{w}_{p1,2} \mathbf{g}_{sk} s_{p1,2}(n) + \sum_{i=1, i\neq k}^{K+1} \mathbf{w}_i \mathbf{h}_{p1} s_i(n) + q_4(n)$$

$$SINR_{PU2} = \frac{\mathbf{w}_{p1,2}^H \mathbf{R}_{p1} \mathbf{w}_{p1,2}}{\sum_{i=1}^{K+1} \mathbf{w}_i^H \mathbf{R}_{s,p1} \mathbf{w}_i + \sigma_n^2}$$

$$SINR_{SUK} = \frac{\mathbf{w}_k^H \mathbf{R}_{sk} \mathbf{w}_k}{\mathbf{w}_{p1,2}^H \mathbf{R}_{p,sk} \mathbf{w}_{p1,2} + \sum_{i=1, i\neq k}^{K+1} \mathbf{w}_i^H \mathbf{R}_{sk} \mathbf{w}_i + \sigma_n^2}$$

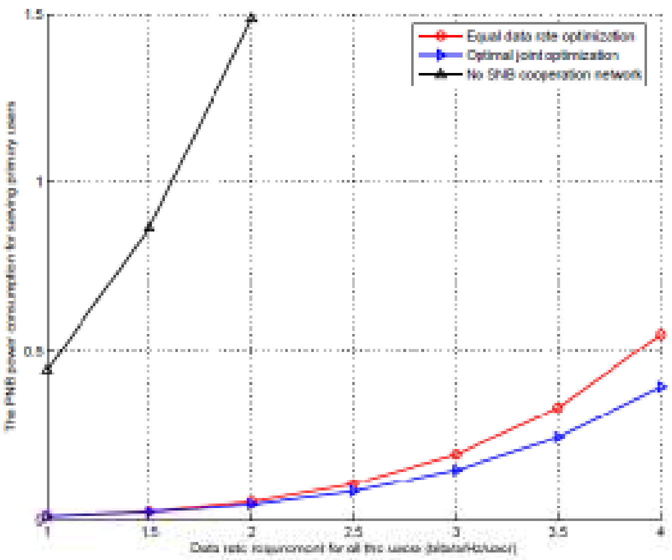
$$\begin{aligned}
\min_{\mathbf{w}_{p1,1}, \mathbf{w}_{ps}, \mathbf{w}_{p1,2}, \mathbf{w}_{k(k=1,\cdots,K+1)}, \delta_{p1,1}, \delta_{p1,2}} \quad & \|\mathbf{w}_{p1,1}\|_2^2 + \|\mathbf{w}_{ps}\|_2^2 + \|\mathbf{w}_{p1,2}\|_2^2 + \sum_{i=1}^{K+1} \|\mathbf{w}_i\|_2^2. \\
\text{s.t.} \quad & SINR_{PU1} \geq \delta_{p1,1}, \\
& SINR_{SNB} \geq \delta_{BS}, \\
& SINR_{PU2} \geq \delta_{p1,2}, \\
& SINR_{SUK} \geq \delta_{sk}, k = 1, 2, \ldots, K+1, \\
& \|\mathbf{w}_{p1,1}\|_2^2 + \|\mathbf{w}_{ps}\|_2^2 \leq P_{p,1}, \\
& \|\mathbf{w}_{p1,2}\|_2^2 \leq P_{p,2}, \\
& \sum_{i=1}^{K+1} \|\mathbf{w}_i\|_2^2 \leq P_s, \\
& \log_2(1 + \delta_{p1,1}) + \log_2(1 + \delta_{p1,2}) = \zeta.
\end{aligned}$$

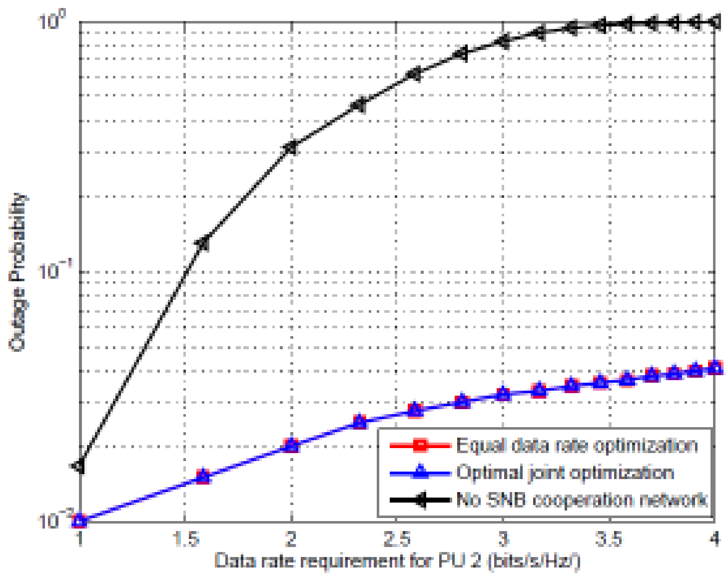
Beamforming for Overlay Cognitive Radio Network



Simulation:
Two primary users and two secondary users.
Multiple antennas at the PNB and SNB

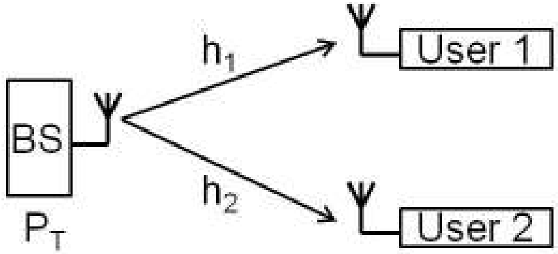
Target rate for all users are varied and the power consumption and the outage probability are observed for the primary network.

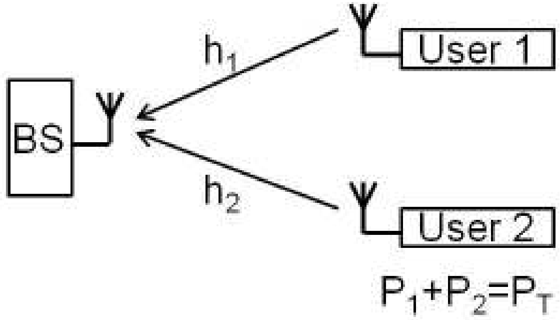




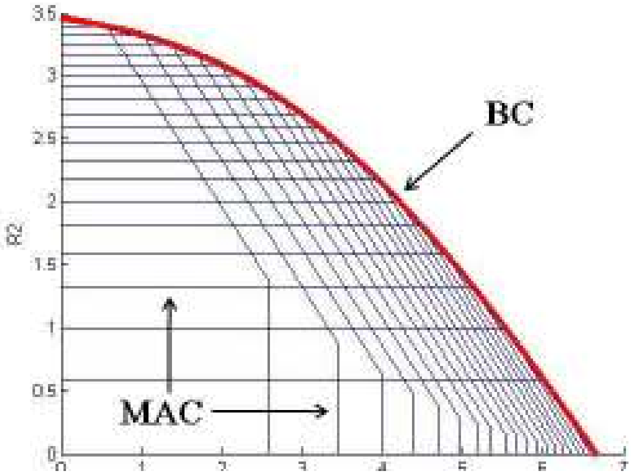
MIMO Spectrum Sharing Channels

We use the results of BC-MAC duality of N.Jindal, S. Vishwanath and A. Goldsmith, IEEE Trans. Information Theory, 2004 to develop weighted sum rate maximization with interference constraints in a MIMO network.





Dual MAC Problem: Both broadcast and multiple access channels share the same capacity region under a sum power constraint.



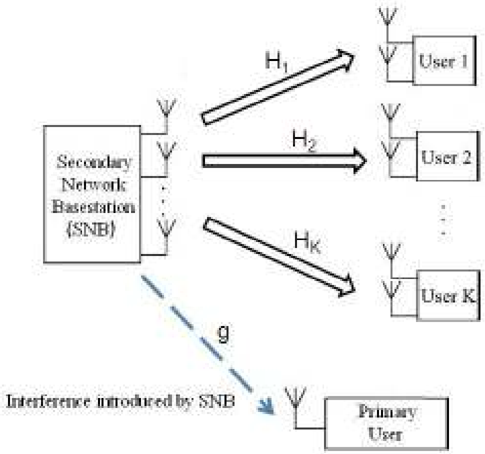
$$\mathbf{P} = (P_1 \quad P_2) \qquad \mathbf{h} = (h_1 \quad h_2) \qquad \bar{P} = P_1 + P_2$$

$$C_{MAC}(\mathbf{P};\mathbf{h}) = \{\mathbf{R} : R_1 + R_2 \leq \log(1 + \frac{1}{\sigma^2}(h_1P_1 + h_2P_2), R_1 \leq \frac{1}{2}\log(1 + \frac{h_1P_1}{\sigma^2}), R_2 \leq \log(1 + \frac{h_2P_2}{\sigma^2})\}.$$

$$C_{BC}(\bar{P};\mathbf{h}) = \{\mathbf{R} : R_1 \leq \frac{1}{2}\log(1 + \frac{h_1P_1}{\sigma^2}), R_2 \leq \frac{1}{2}\log(1 + \frac{h_2P_2}{\sigma^2 + h_1P_1})\}$$

MIMO Spectrum Sharing Channels

Weighted sum rate maximization with interference constraints in MIMO-OFDM network



Original rate maximization problem in broadcast channels

$$\min_{\alpha\geq 0,\beta\geq 0}\ \max_{\{\mathbf{Q}^b_{n,k}\}k=1,\cdots,K:\mathbf{Q}^b_{n,k}\succeq 0}\ \sum_{k=1}^{K}\sum_{n=1}^{N}\mu_k R^b_{n,k}$$

$$\text{s.t.}\quad \alpha\Big(\sum_{k=1}^{K}\sum_{n=1}^{N} tr(\mathbf{Q}^b_{n,k}) - P_t\Big)$$

$$+\beta\Big(\sum_{k=1}^{K}\sum_{n=1}^{N}\mathbf{h}^H_{n,0}\mathbf{Q}^b_{n,k}\mathbf{h}_{n,0} - P_I\Big) \leq 0,$$

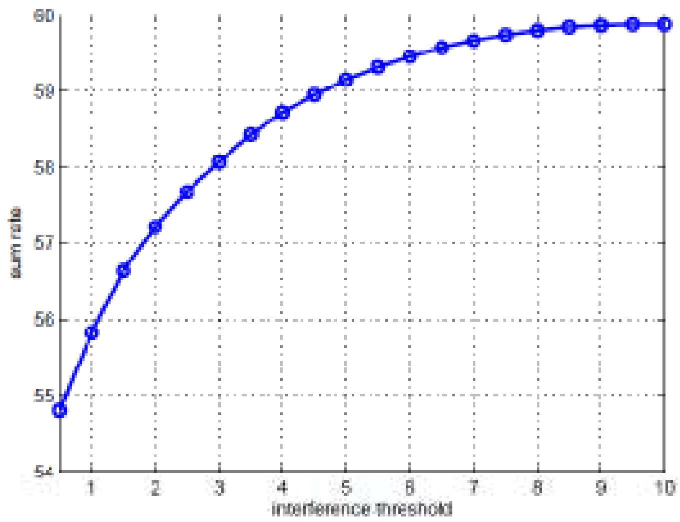
Dual MAC Problem

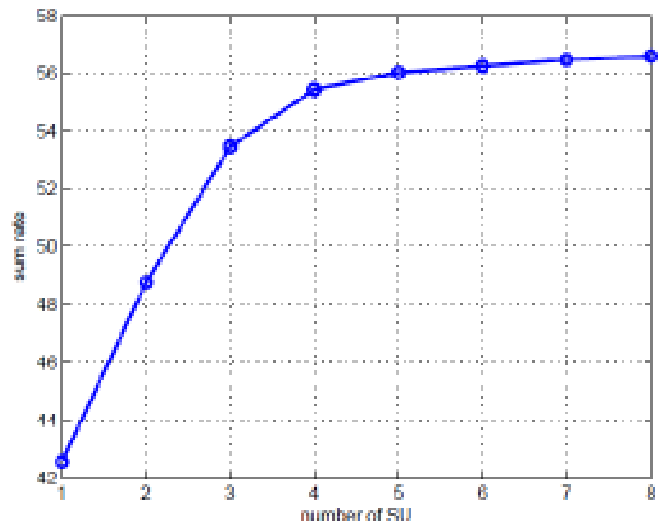
$$\max_{\{\mathbf{Q}^m_{n,k}\}k=1,\cdots,K.\ n=1,\cdots,N\ :\mathbf{Q}^m_{n,k}\succeq 0}\ \sum_{i=1}^{K}\sum_{n=1}^{N}\mu_k R^m_{n,k}$$

$$\text{s.t.}\quad \sum_{k=1}^{K}\sum_{n=1}^{N} tr(\mathbf{Q}^m_{n,k})\sigma^2 \leq \alpha P_t + \beta P_I,$$

MIMO Spectrum Sharing Channels

Weighted sum rate maximization with interference constraints in MIMO-OFDM network



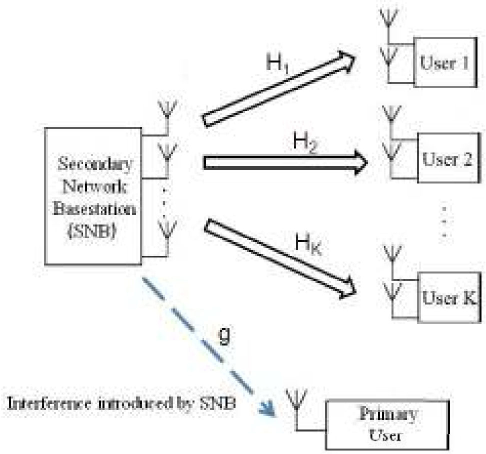


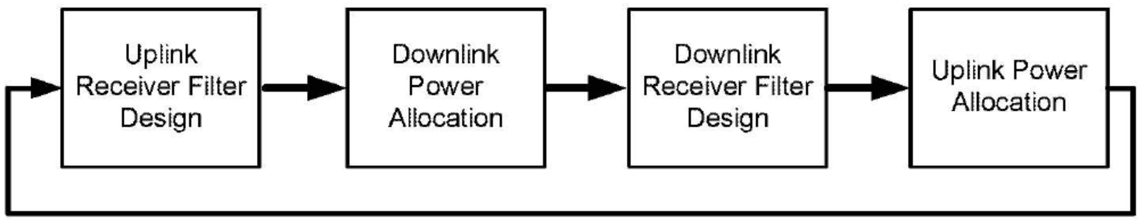
Simulation: Five Transmitting Antennas and Five Users each with three antennas.
One primary user with a single receiver antenna.
The Transmission power is 20W and the noise power at the receiver is 1W.

For Figure 2: The interference threshold is one

Multiuser MIMO Transceiver Design with Interference Constraints

Optimization of Transmission Power subject to Individual data Constraints and Multiple Interference Leakage constraints.

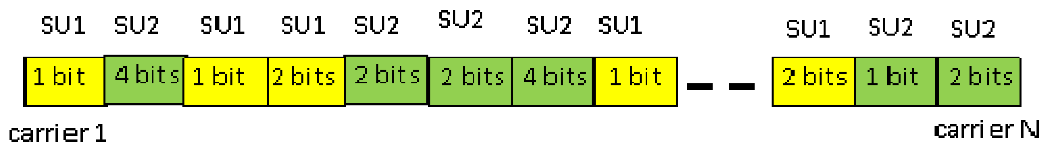




K. Cumanan, Y. Rahulamathavan and S. Lambotharan, ICWR 2010, Bangalore.

Adaptive Power and Bit Loading in OFDMA based CRN.

- There are K secondary users and L primary users and N subcarriers.
- Allocate the subcarriers and the transmission power optimally to the secondary users while ensuring interference leakage to primary users is below a threshold.



OFDMA Resource Allocation Scheme: Power and Interference Constraints

The aim is to maximize the total throughput subject to various constraints as

$$
\begin{aligned}
\max_{c_{k,n}\in\{0,1,2,4,6\}} \quad & \sum_{n=1}^{N}\sum_{k=1}^{K} c_{k,n}, \\
\text{s.t.} \quad & \sum_{n=1}^{N} \phi_n \beta_{ln}^2 \leq \xi_l, \quad l = 1, 2, \ldots, L, \\
& \sum_{n=1}^{N} c_{k,n} \geq r_k, \quad k = 1, 2, \ldots, K, \\
& \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \leq P, \\
& c_{k,n} = 0 : c_{k',n} \neq 0, \\
& \forall k \neq k', \quad (k = 1, 2, \ldots, K),
\end{aligned}
$$

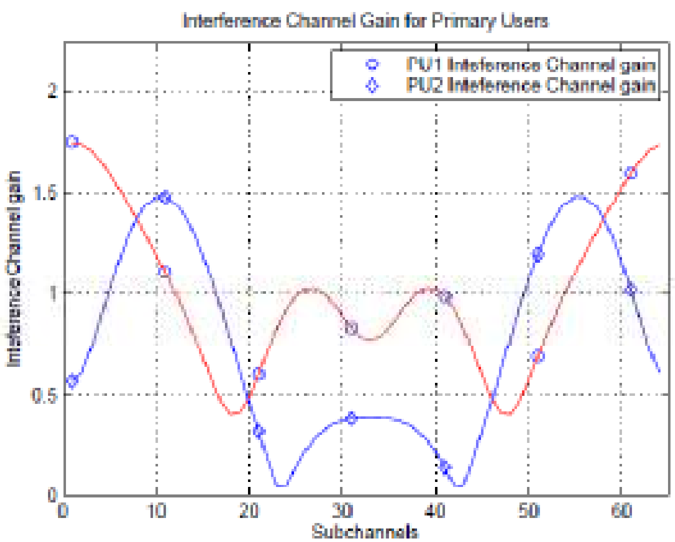
$c_{k,n}$ - the number of bits allocated for k^{th} SU over the n^{th} subcarrier.
$\phi_n = \sum_{k=1}^{K} P_{k,n}$ - the total power allocated in the n^{th} subcarrier
$P_{k,n}$- power allocated for the k^{th} SU in the n^{th} subcarrier
P - the maximum total transmission power

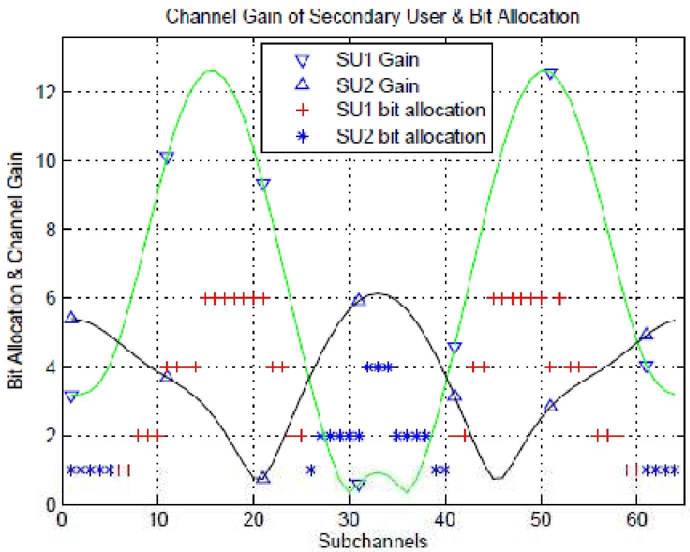
Integer Linear Programming

Using appropriate definitions of other matrices, the original optimization problem can be given in linear integer programming form as follows

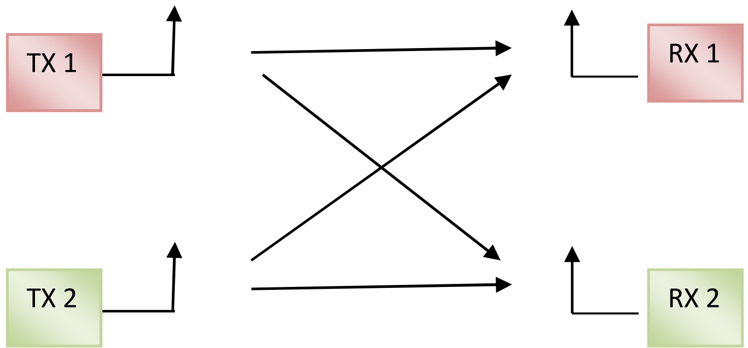
$$
\begin{aligned}
\max_{c_{k,n}\in\{0,1,2,4,6\}} \quad & \sum_{n=1}^{N}\sum_{k=1}^{K} c_{k,n}, \\
\text{s.t.} \quad & \sum_{n=1}^{N} \phi_n \beta_{ln}^2 \le \xi_l, \quad l = 1,2,\ldots,L, \\
& \sum_{n=1}^{N} c_{k,n} \ge r_k, \quad k = 1,2,\ldots,K, \\
& \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \le P, \\
& c_{k,n} = 0 : c_{k',n} \ne 0, \\
& \forall k \ne k', \quad (k = 1,2,\ldots,K),
\end{aligned}
$$

$$
\begin{aligned}
\min_{x} \quad & -\mathbf{b}^T\mathbf{x} \\
\text{s.t.} \quad & \mathbf{H_i}[\mathbf{A}_c(\mathbf{p}\odot\mathbf{x})] \le \xi_i \\
& \mathbf{A}_u\mathbf{x} \ge \mathbf{R} \\
& \mathbf{p}^T\mathbf{x} \le P \\
& \mathbf{0}_N \le \mathbf{A}_c\mathbf{x} \le \mathbf{1}_N \\
& x_i \in \{0,1\}, \quad i = 1,2,\ldots,NKC
\end{aligned}
$$





Non-cooperative Games in Spectrum Sharing Channels



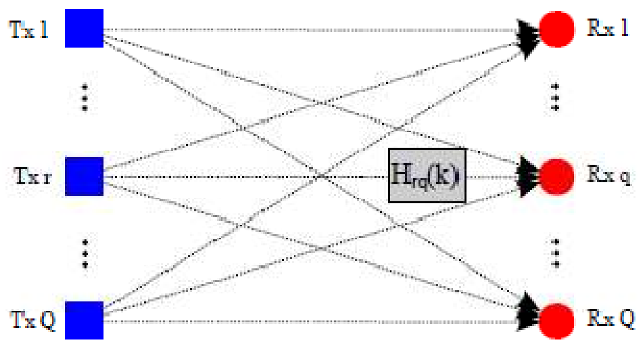
$$R1 = \log_2 (1 + \frac{P_1 |h_{11}|^2}{P_2 |h_{21}|^2 + \sigma_n^2})$$

$$R2 = \log_2 (1 + \frac{P_2 |h_{22}|^2}{P_1 |h_{12}|^2 + \sigma_n^2})$$

Gaussian Interference Channel Model

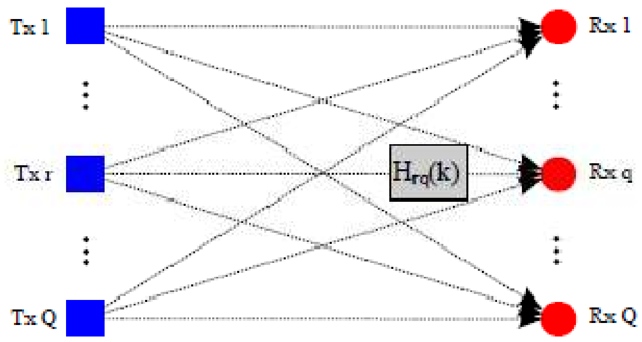
- ❑ Players (TX1 and TX2) compete for resources.
- ❑ Players are assumed to be rational, but non-cooperative.
- ❑ The strategy space is the power allocation p1 and p2.
- ❑ Utility functions are R1 and R2.
- ❑ Non-cooperative games settle for Nash equilibrium.
- ❑ Nash equilibrium is an operating point where no player can change its strategy unilaterally to improve its utility.

Robust Rate-Maximization Game Under Bounded Channel Uncertainty



- Frequency-selective Gaussian interference channel with N frequencies, composed of Q SISO links
- $H_{rq}(k)$ denotes the frequency response of the k-th frequency bin of the channel between source r and destination q
- $\mathbf{p}_q \triangleq [p_q(1)\ p_q(2) \ldots p_q(N)]$ is the power allocation vector of user q
- Constraints on power allocation:
  - Total power for each user: $\frac{1}{N}\sum_{k=1}^{N} p_q(k) \leq 1$
  - Spectral mask: $p_q(k) \leq p_q^{max}(k)$

Robust Rate-Maximization Game Under Bounded Channel Uncertainty



The information rate of user q can be written as

$$R_q = \sum_{k=1}^{N} \log\left(1 + \frac{p_q(k)}{\sigma_q^2(k) + \sum_{r\neq q} F_{rq}(k) p_r(k)}\right)$$

where $F_{rq}(k) \triangleq \frac{|H_{rq}(k)|^2}{|H_{qq}(k)|^2}$

Uncertainty in CSI is modelled as ellipsoid approximation

$$\mathcal{F}_q = \left\{ F_{rq}(k) + \Delta F_{rq,k} \ : \ \sum_{r\neq q} |\Delta F_{rq,k}|^2 \leq \epsilon_q^2 \right\}$$

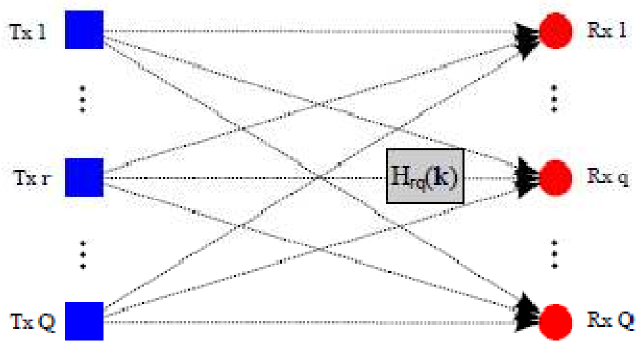
Robust Rate-Maximization Game Under Bounded Channel Uncertainty

Mathematically, the nominal game $\mathscr{G}^{\text{nom}}$ can be written as

$$\begin{aligned} &\max_{\mathbf{p}_q} \quad \sum_{k=1}^{N} \log\left(1+\frac{p_q(k)}{\sigma_q^2(k)+\sum_{r\neq q} F_{rq}(k)p_r(k)}\right) \qquad \forall q \in \Omega, \\ &\text{s.t.} \quad \mathbf{p}_q \in \mathcal{P}_q \end{aligned}$$

where $\Omega \triangleq \{1,\ldots,Q\}$ is the set of the Q players (i.e. the SISO links) and $\mathcal{P}_q$ is the set of admissible strategies of user q, which is defined as

$$\mathcal{P}_q \triangleq \left\{\mathbf{p}_q \in \mathbb{R}^N : \frac{1}{N}\sum_{k=1}^{N} p_q(k) = 1,\ 0 \le p_q(k) \le p_q^{max}(k),\ k = 1,\ldots,N\right\}.$$



we get the robust game $\mathscr{G}^{\text{rob}}$ as, $\quad \forall q \in \Omega,$

$$\begin{aligned} &\max_{\mathbf{p}_q} \sum_{k=1}^{N} \log\left(1+\frac{p_q(k)}{\sigma_q^2(k)+\sum_{r\neq q} F_{rq}(k)p_r(k)+\epsilon_q\sqrt{\sum_{r\neq q} p_r^2(k)}}\right) \\ &\text{s.t.} \quad \mathbf{p}_q \in \mathcal{P}_q. \end{aligned}$$

Robust Rate-Maximization Game Under Bounded Channel Uncertainty

Theorem 1. *Given the set of power allocations of other users* $\mathbf{p}_{-q} \triangleq \{\mathbf{p}_1, \ldots, \mathbf{p}_{q-1}, \mathbf{p}_{q+1}, \ldots, \mathbf{p}_Q\}$, *the solution to the robust optimization problem of user* q,

$$\max_{\mathbf{p}_q} \sum_{k=1}^{N} \log\left(1 + \frac{p_q(k)}{\sigma_q^2(k) + \sum_{r \neq q} F_{rq}(k) p_r(k) + \epsilon_q \sqrt{\sum_{r \neq q} p_r^2(k)}}\right)$$

$$\text{s.t.} \quad \mathbf{p}_q \in \mathcal{P}_q.$$

is given by the waterfilling solution

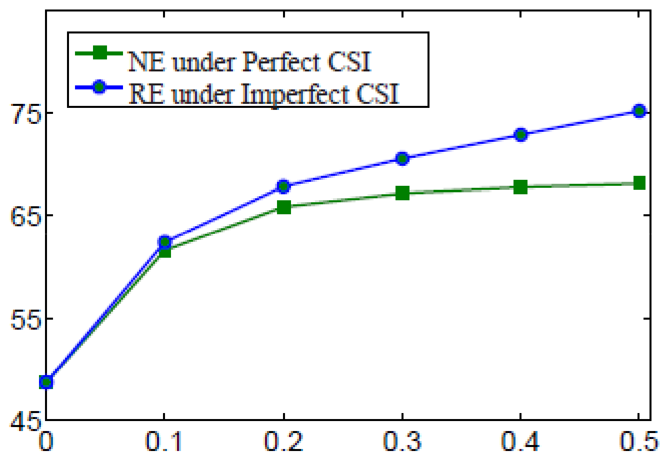
$$\mathbf{p}_q^\star = \mathrm{RWF_q}(\mathbf{p}_{-q}),$$

where the waterfilling operator $\mathrm{RWF_q}(\cdot)$ *is defined as*

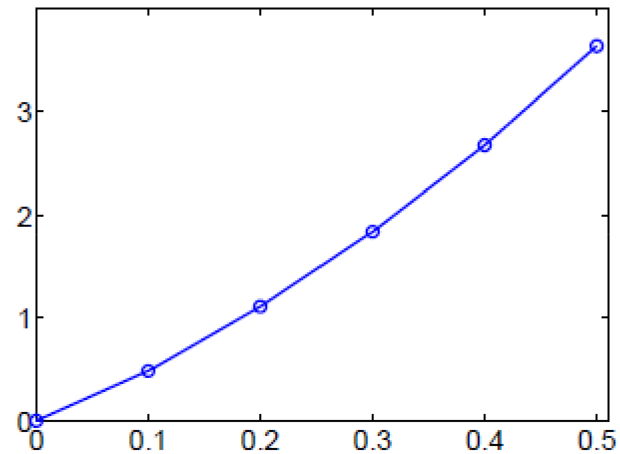
$$[\mathrm{RWF_q}(\mathbf{p}_{-q})]_k \triangleq \left[\mu_q - \sigma_q^2(k) - \sum_{r \neq q} F_{rq}(k) p_r(k) - \epsilon_q \sqrt{\sum_{r \neq q} p_r^2(k)}\right]_0^{p_q^{max}(k)}$$

for $k = 1, \ldots, N$, *where* μ_q *is chosen to satisfy the power constraint* $\sum_{k=1}^{N} p_q^\star(k) = 1$.

Simulation Results



(b) Total rate observed at rx. vs. uncertainty δ.



(c) Extra no. of zero alloc. for RE vs. uncertainty δ.

Loughborough University

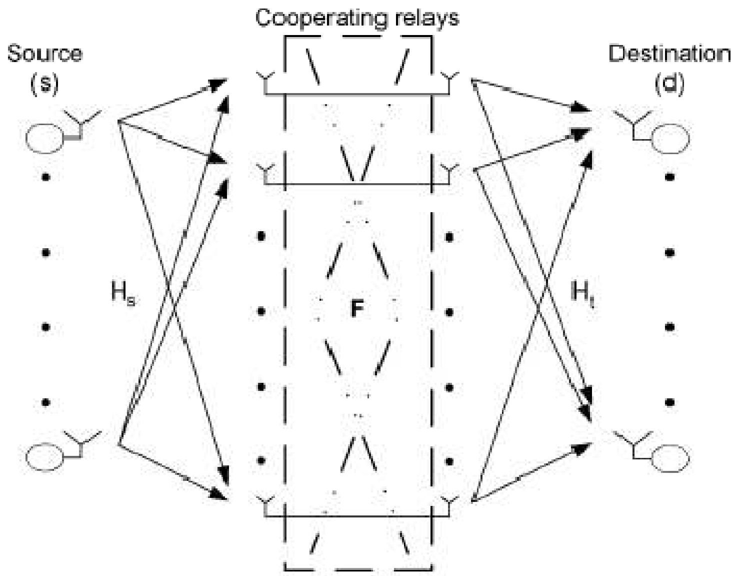
Other Related Research Activities

1) Joint Downlink Beamforming and User Admission Control

K. Cumanan, R. Krishna, L. Musavian and S. Lambotharan, "Joint Beamforming and User Maximization Techniques for Cognitive Radio Networks Based on Branch and Bound Method," in IEEE Trans. Wireless Communications, 2010

2) Optimal Signal Forwarding Techniques for Relay Networks

R. Krishna, K. Cumanan, Z. Xiong and S. Lambotharan, "A Novel Cooperative Relaying Strategy For Wireless Networks With Signal Quantization," IEEE Trans. Vehicular Technology, Jan. 2010



3) Effective Capacity of Fading Channels for Relay and Cognitive Radio Networks

L. Musavian, S. Aïssa and S. Lambotharan, "Effective Capacity for Interference and Delay Constrained Cognitive Radio Relay Channels", IEEE Trans. Wireless Communications, vol. 9 (5), pp. 1698-1707, May 2010.
L. Musavian, S. Aïssa and S. Lambotharan, "Adaptive Modulation in Spectrum-Sharing Channels Under Delay Quality of Service Constraints," accepted for publication in IEEE Transactions Vehicular Technology, Oct. 2010.

Thank You.